Analysis of Algorithms on Growing Networks

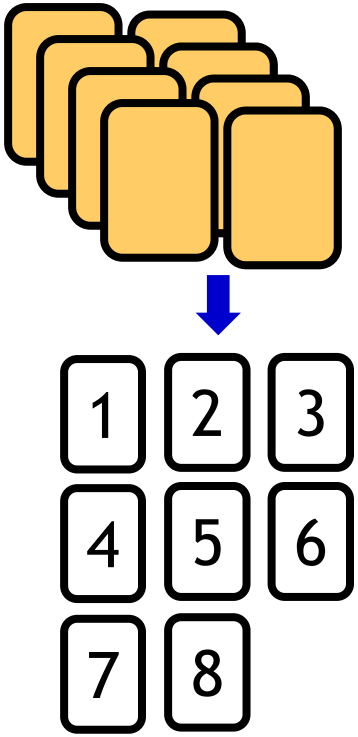
--- Can you correct all of *an increasing number of* coupons?

*Shuji Kijima (Shiga Univ.)

1. Coupon Collector’s Problem

Collecting Coupons (a.k.a. Coupon collector's problem)

- ✓ n cards are laid face-down on a desk, where the faces are marked with distinguishing motifs.
- ✓ You will repeat trials, each of which consists of choosing a card, checking its face, putting it back face-down, and mixing cards completely (i.e., sampling with replacement).
- ✓ How many times do you need to repeat the trials to see all n motifs?



Ex.

- P●KEM●N trading cards
- Bikkuriman
- Capsule toys

Collecting Coupons

- ✓ $X_t \in \{1, \dots, n\}$ $(t = 1,2, \dots)$ independent, $\Pr[X_t = k] = \frac{1}{n}$ for $k \in \{1, \dots, n\}$.
- ✓ $T = \min\{t' \mid \{X_1, \dots, X_{t'}\} = \{1, \dots, n\}\}$: completion time.

Q. Find $\mathrm{E}[T]$.

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Let $T_k \coloneqq \min\{t \mid |\{X_1, \ldots, X_t\}| = k\}$ for $k = 1,2, \ldots, n$.

Let $S_k \coloneqq T_k - T_{k-1}$.

Claim. $\mathrm{E}[S_k] = \frac{n}{n-k+1}$.

✓ Prob. on cond. $k-1$: $p_k = \frac{n-(k-1)}{n}$.

✓ Thus $\mathrm{E}[S_k] = \frac{1}{p_k} = \frac{n}{n-k+1}$ (Geom. Distr.).

Since $T = T_n = \sum_{k=1}^{n} S_k$,

$$\mathrm{E}[T] = \mathrm{E}[T_n] = \sum_{k=1}^{n} \mathrm{E}[S_k] = \sum_{k=1}^{n} \frac{n}{n-k+1} = n \sum_{k'=1}^{n} \frac{1}{k'} \simeq n(\ln n + 0.577).$$

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Q. Find $\Pr[T \geq 2n \ln n]$.

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By Chevyshev's inequality,

$$\Pr\left[X \geq 2\mathrm{E}[X]\right] \leq \frac{\mathrm{Var}[X]}{\mathrm{E}[X]^2} \leq \frac{\frac{n^2\pi^2}{6}}{(n \ln n)^2} = \frac{\pi^2}{6(\ln n)^2}$$

$\simeq 0.078$ (for $n = 100$)

Improved by Hoefding ineq., Chernoff ineq., etc.

2. Collecting coupons

an increasing number of

Collecting *an increasing number of* coupons

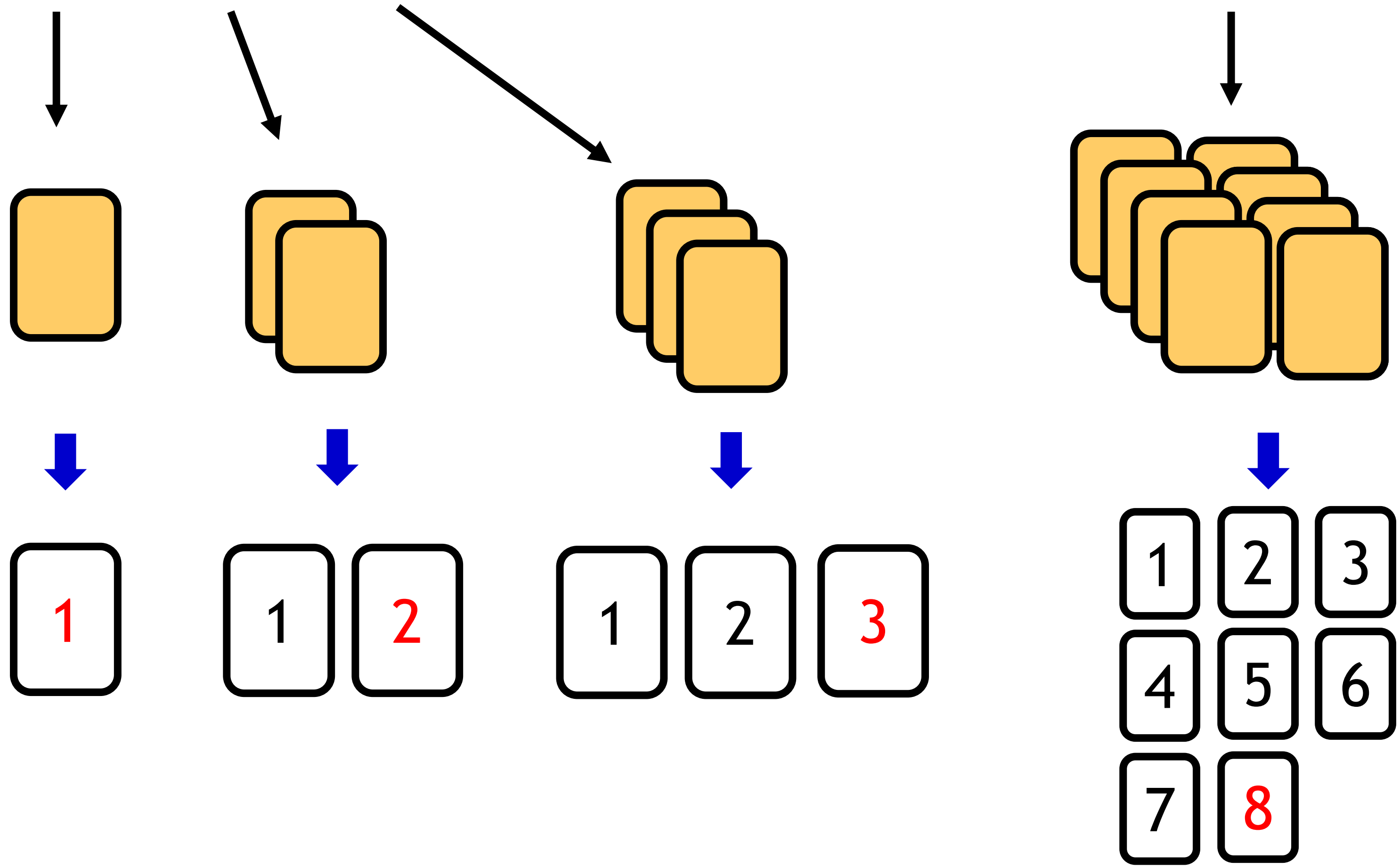
Draw a coupon everyday, u.a.r. from a finite #types.

- A new type of coupons is released everyday. I.e., you draw a coupon from d types on the d^{th} day. **When can you complete?**

"complete"
= collect all types.

Collecting *an increasing number of* coupons

Day (t)	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9
# types	1	2	3	4	5	6	7	8	9
$\Pr[X_t = k]$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$



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- A new type is released every two days. I.e., you draw from $\left\lceil \frac{d}{2} \right\rceil$ on the d^{th} day. **When can you complete?**

Collecting *an increasing number of* coupons

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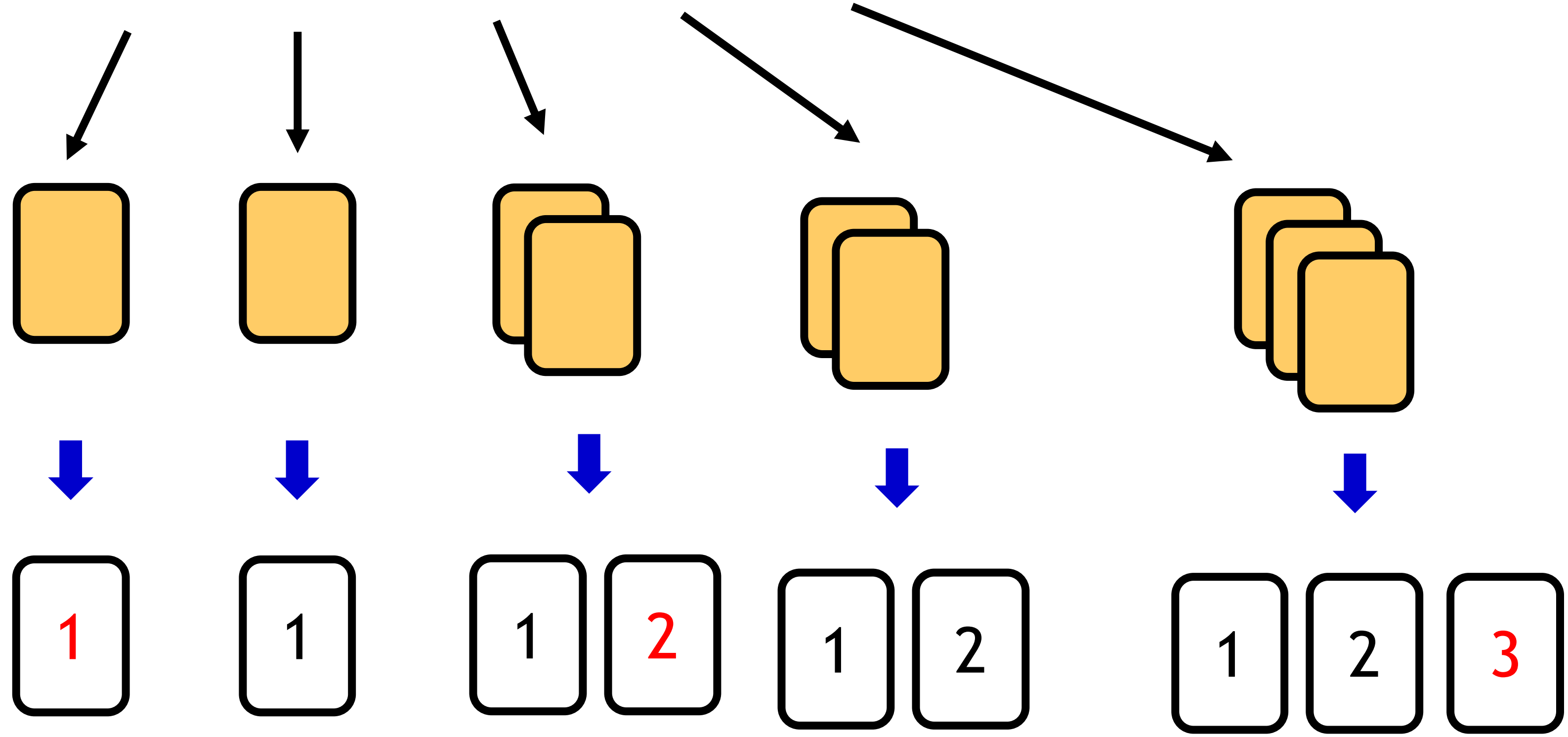
# types	1	1	2	2	3	3	4	4	5
$\Pr[X_t = k]$	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{5}$



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  - ➢ **Impossible**, maybe.
- A new type is released every 10 days. I.e., you draw from $\left\lceil \frac{d}{10} \right\rceil$ on the d^{th} day. **When can you complete?**

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- A new type is released every 10 days. I.e., you draw from $\left\lceil \frac{d}{10} \right\rceil$ on the d^{th} day. **When can you complete?**
  - **Impossible**. (Consider the 1000^{th} day. Can you draw the new one from 100 types in 10 days?)

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  - ➢ **Impossible**. (Consider the 1000^{th} da… draw the new one from 100 types in 1…

Not very fun…

Collecting *a* **moderately** *increasing number of* coupons

Draw a coupon everyday, u.a.r. from a finite #types.

- The n^{th} type of coupon is released n days after $n-1^{\text{th}}$ was released. **When can you complete?**

Collecting *a* **moderately** *increasing number of* coupons

Day	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9
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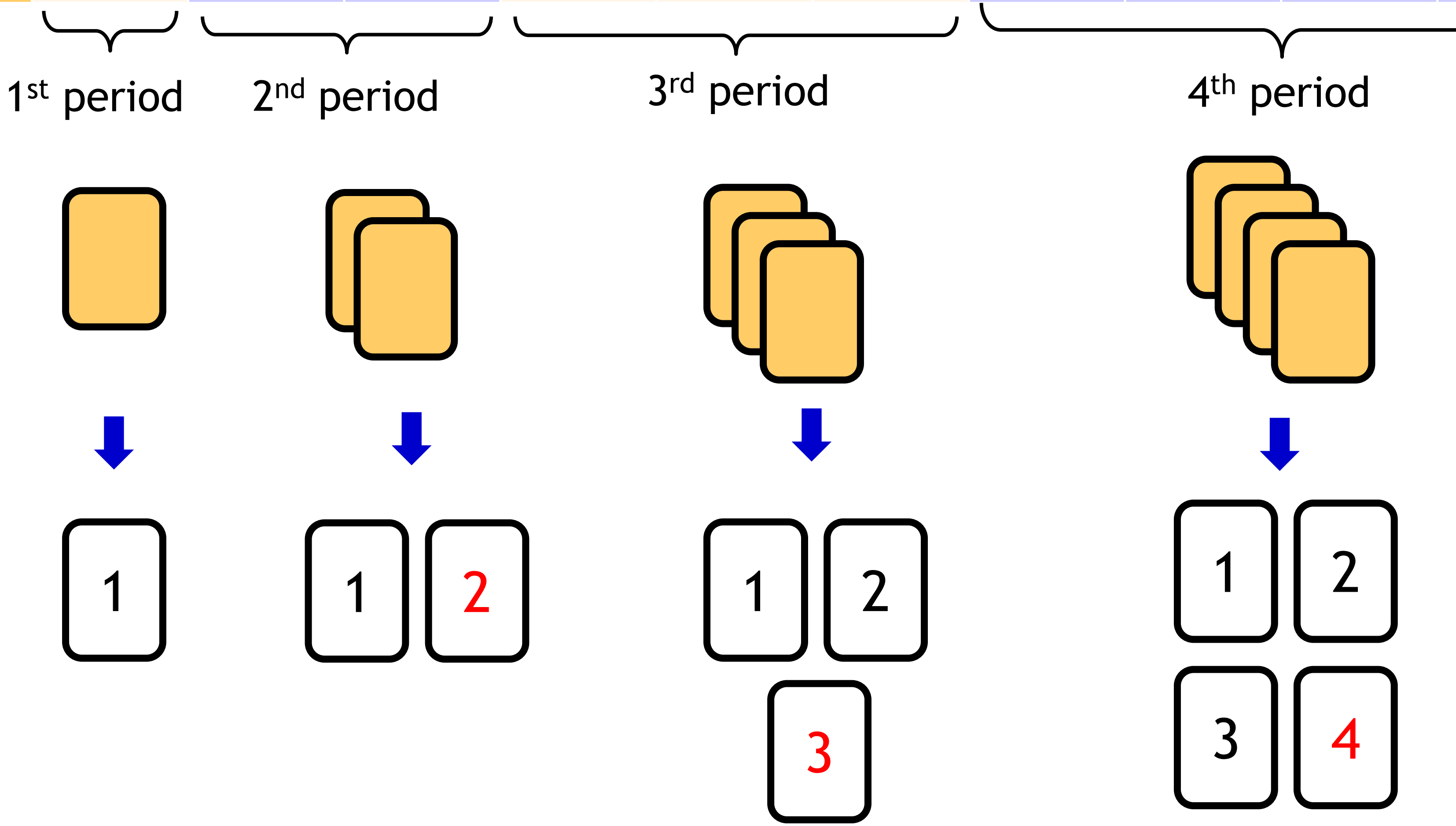
$\Pr[X_t = k]$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$



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  - Q. How many types are collected in the end of n^{th} period?
    1. $\mathrm{o}(n)$
    2. cn
    3. $n - c\sqrt{n}$
    4. $n - c \log n$
    5. $n - c$

c is some constant.

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  - Q. How many types are collected in the end of n^{th} period?
    1. $o(n)$
    2. cn
    3. $n - c\sqrt{n}$
    4. $n - c\log n$
    5. **$n - c$ (right answer)**

c is some constant.

Collecting *a* **moderately** *increasing nu*

Draw a coupon everyday

$\mathfrak{d}(n)$: #days of the n^{th} period

U_n: #items uncollected in the end of n^{th} period

Prop.

If $\mathfrak{d}(n) = n$ then $\mathrm{E}[U_n] < 1$.

Collecting *a* **moderately** *increasing number of* coupons

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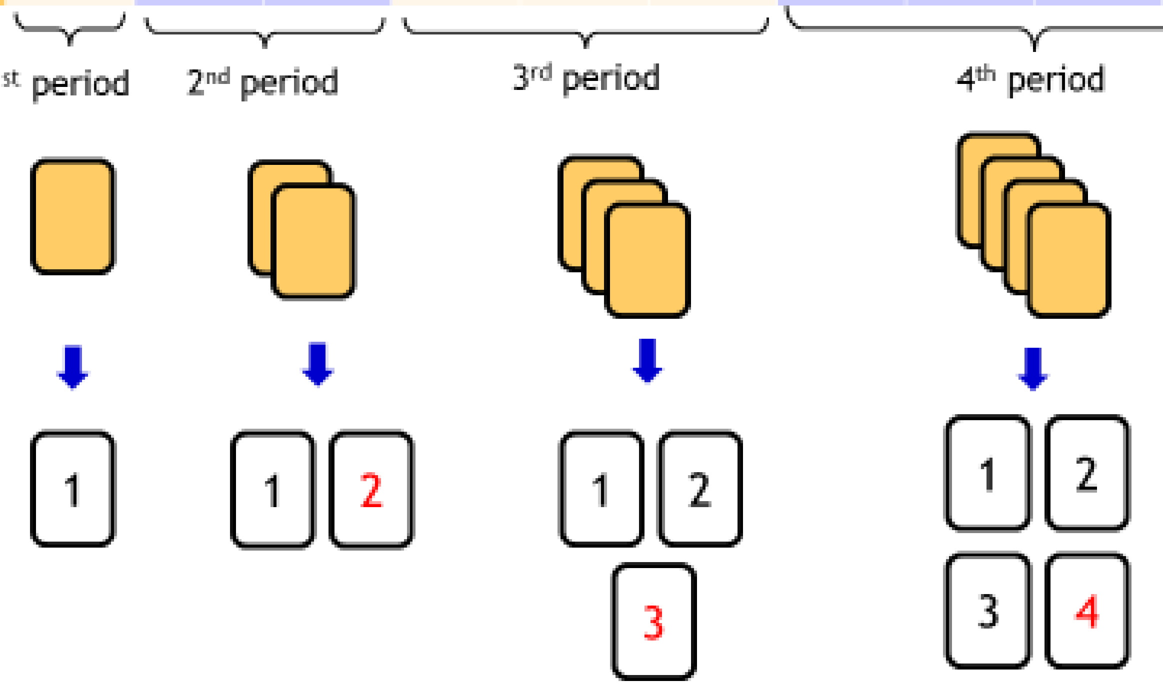
$\Pr[X_i = k]$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$



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If $\mathfrak{d}(n) = n$ then $\mathrm{E}[U_n] < 1$.

Proof.

- ✓ $\mathcal{E}_{i,n} := \begin{cases} 1 & \text{(item } i \text{ is uncollected in the end of the } n^{\text{th}} \text{ period)} \\ 0 & \text{(item } i \text{ is collected by the end of the } n^{\text{th}} \text{ period)} \end{cases}$

  for $i = 1,2,\dots,n$.
- ✓ $U_n = \sum_{i=1}^{n} \mathcal{E}_{i,n}$
- ✓ Prob. that item n is uncollected in the end of the nth period:

$$\Pr[\mathcal{E}_{n,n} = 1] = \left(1 - \frac{1}{n}\right)^n < \mathrm{e}^{-1}$$

- ✓ Prob. that item i $(i \le n)$ is uncollected in the end of the n^{th} period:

$$\Pr[\mathcal{E}_{i,n} = 1] = \left(1 - \frac{1}{i}\right)^i \left(1 - \frac{1}{i+1}\right)^{i+1} \dots \left(1 - \frac{1}{n}\right)^n < \left(\frac{1}{\mathrm{e}}\right)^{n+1-i}$$

- ✓ $\mathrm{E}[U_n] = \sum_{i=1}^{n} \Pr[\mathcal{E}_{i,n}] < \sum_{i=1}^{n} \left(\frac{1}{\mathrm{e}}\right)^{n+1-i} = \frac{1}{\mathrm{e}} + \frac{1}{\mathrm{e}^2} + \cdots + \frac{1}{\mathrm{e}^n} < \frac{\frac{1}{\mathrm{e}}}{1-\frac{1}{\mathrm{e}}} = \frac{1}{\mathrm{e}-1} < 0.582.$

Collecting *a* **moderately** *increasing nu*

Draw a coupon everyday

$\mathfrak{d}(n)$: #days of the n^{th} period

U_n: #items uncollected in the end of n^{th} period

Prop.

If $\mathfrak{d}(n) = \frac{n}{2}$ then $\mathrm{E}[U_n] < ???$.

Collecting *a* **moderately** *increasing nu*

Draw a coupon everyday

$\mathfrak{d}(n)$: #days of the n^{th} period

U_n: #items uncollected in the end of n^{th} period

Prop.

If $\mathfrak{d}(n) = \frac{n}{2}$ then $\mathrm{E}[U_n] < 2$.

Proof.

- ✓ $\mathcal{E}_{i,n} := \begin{cases} 1 & (\text{item } i \text{ is uncollected in the end of the } n^{\text{th}} \text{ period}) \\ 0 & (\text{item } i \text{ is collected by the end of the } n^{\text{th}} \text{ period}) \end{cases}$

  for $i = 1,2, \ldots, n$.
- ✓ $U_n = \sum_{i=1}^{n} \mathcal{E}_{i,n}$
- ✓ Prob. that item n is uncollected in the end of the nth period:

$$\Pr[\mathcal{E}_{n,n} = 1] = \left(1 - \frac{1}{n}\right)^{\frac{n}{2}} < \mathrm{e}^{-\frac{1}{2}}$$

- ✓ Prob. that item i $(i \leq n)$ is uncollected in the end of the n^{th} period:

$$\Pr[\mathcal{E}_{i,n} = 1] = \left(1 - \frac{1}{i}\right)^{\frac{i}{2}} \left(1 - \frac{1}{i+1}\right)^{\frac{i+1}{2}} \ldots \left(1 - \frac{1}{n}\right)^{\frac{n}{2}} < \left(\frac{1}{\mathrm{e}}\right)^{\frac{n+1-i}{2}}$$

- ✓ $\mathrm{E}[U_n] = \sum_{i=1}^{n} \Pr[\mathcal{E}_{i,n}] < \sum_{i=1}^{n} \left(\frac{1}{\mathrm{e}}\right)^{\frac{n+1-i}{2}} = \frac{1}{\mathrm{e}^{\frac{1}{2}}} + \frac{1}{\mathrm{e}^{\frac{2}{2}}} + \cdots + \frac{1}{\mathrm{e}^{\frac{n}{2}}} < \frac{\frac{1}{\mathrm{e}^{\frac{1}{2}}}}{1 - \frac{1}{\mathrm{e}^{\frac{1}{2}}}} = \frac{1}{\mathrm{e}^{\frac{1}{2}} - 1} < 1.542.$

Collecting a **moderately** *increasing nu*

Draw a coupon everyday

$\mathfrak{d}(n)$: #days of the n^{th} period

U_n: #items uncollected

in the end of n^{th} period

Prop.

If $\mathfrak{d}(n) = cn$ then $\mathrm{E}[U_n] < \frac{1}{\mathrm{e}^c - 1}$.

Proof.

✓ Prob. that item i $(i \le n)$ is uncollected in the end of the n^{th} period:

$$\Pr[\mathcal{E}_{i,n} = 1] = \left(1 - \frac{1}{i}\right)^{ci} \left(1 - \frac{1}{i+1}\right)^{c(i+1)} \ldots \left(1 - \frac{1}{n}\right)^{cn} \le \left(\frac{1}{\mathrm{e}}\right)^{c(n+1-i)}$$

✓ $\mathrm{E}[U_n] = \sum_{i=1}^{n} \Pr[\mathcal{E}_{i,n}] \le \sum_{i=1}^{n} \left(\frac{1}{\mathrm{e}}\right)^{c(n+1-i)} = \frac{1}{\mathrm{e}^c} + \frac{1}{\mathrm{e}^{2c}} + \cdots + \frac{1}{\mathrm{e}^{nc}} < \frac{\frac{1}{\mathrm{e}^c}}{1 - \frac{1}{\mathrm{e}^c}} = \frac{1}{\mathrm{e}^c - 1}$

Collecting *a* **moderately** *increasing nu*[...]

Draw a coupon everyday

$\mathfrak{d}(n)$: #days of the n^{th} period

U_n: #items uncollected in the end of n^{th} period

Prop. (lower bound)

If $\mathfrak{d}(n) = cn$ then $\mathrm{E}[U_n] \geq \frac{n}{cn+1}\left(1 - \frac{1}{n}\right)^{cn} \simeq \frac{1}{ce^c}$.

Proof.

- ✓ $S(n) := \sum_{k=1}^{n} \prod_{i=k}^{n} \left(1 - \frac{1}{i}\right)^{ci}$ $\quad (= \mathrm{E}[U_n])$
- ✓ Proposition: $S(n) \geq \frac{n}{cn+1}\left(1 - \frac{1}{n}\right)^{cn}$.
  - Claim 1: $S(n+1) = \left(1 - \frac{1}{n+1}\right)^{c(n+1)} (S(n) + 1)$.
  - Claim 2: $S(n) \geq \frac{n}{cn+1}\left(1 - \frac{1}{n}\right)^{cn} \Rightarrow S(n) + 1 \geq \frac{n+1}{c(n+1)+1}$.
  - Inductively,

$$S(n+1) = \left(1 - \frac{1}{n+1}\right)^{c(n+1)} (S(n) + 1)$$

$$\geq \left(1 - \frac{1}{n+1}\right)^{c(n+1)} \frac{n+1}{c(n+1)+1}$$

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Proof.

✓ $S(n) \coloneqq \sum_{k=1}^{n} \prod_{i=k}^{n} \left(1-\frac{1}{i}\right)^{ci}$ $\quad (= \mathrm{E}[U_n])$

✓ Proposition: $S(n) \geq \frac{n}{cn+1}\left(1-\frac{1}{n}\right)^{cn}$.

- Claim 1: $S(n+1) = \left(1-\frac{1}{n+1}\right)^{c(n+1)} (S(n)+1)$.
- Claim 2: $S(n) \geq \frac{n}{cn+1}\left(1-\frac{1}{n}\right)^{cn} \Rightarrow S(n)+1 \geq \frac{n+1}{c(n+1)+1}$.
- Inductively,

$$S(n+1) = \left(1-\frac{1}{n+1}\right)^{c(n+1)} (S(n)+1)$$

$$\geq \left(1-\frac{1}{n+1}\right)^{c(n+1)} \frac{n+1}{c(n+1)+1}$$

Collecting *a* **moderately** *increasing number of* coupons

Lemma (about S_n)

$\mathfrak{d}\colon \mathbb{N} \to \mathbb{N}$, $S(n) \coloneqq \sum_{k=1}^{n} \prod_{i=k}^{n} \left(1 - \frac{1}{i}\right)^{\mathfrak{d}(i)}$

(i) If $\mathfrak{d}(i) \geq ci$ then $S(n) = O(1)$.

(ii) If $\mathfrak{d}$ is monotone nonincreasing ($\mathfrak{d}(i) \leq \mathfrak{d}(i+1)$) then

$$S(n) \geq \frac{n}{\mathfrak{d}(n)+1}\left(1 - \frac{1}{n}\right)^{\mathfrak{d}(n)}.$$

(iii) If $\mathfrak{d}$ is sublinear ($\frac{\mathfrak{d}(i)}{i} \geq \frac{\mathfrak{d}(i+1)}{i+1}$) then $S(n) \leq \frac{n}{\mathfrak{d}(n)}$.

(iv) If $\mathfrak{d}(i) = c\ (\text{const.})$ then $S(n) \leq \frac{n}{c+1}$.

Collecting *an increasing number of* cou

Draw a coupon everyday

$\mathfrak{d}(n)$: #days of the n^{th} period

U_n: #items uncollected in the end of n^{th} period

Thm.

$\mathfrak{d}: \mathbb{N} \to \mathbb{N}$.

(i) If $\mathfrak{d}(i) \geq ci$ then $\mathrm{E}[U_n] = \mathrm{O}(1)$.

- If $\frac{\mathfrak{d}(i)}{i} \xrightarrow{i\to\infty} \infty$ then $\mathrm{E}[U_n] = 0$.

(ii) If $\mathfrak{d}$ is monotone nondecreasing, unbounded and sublinear ($\frac{\mathfrak{d}(i)}{i} \geq \frac{\mathfrak{d}(i+1)}{i+1}$) then

$$E[U_n] = \left(1 - \mathrm{o}(1)\right)\frac{n}{\mathfrak{d}(n)+1}.$$

(iii) If $\mathfrak{d}(i) = c$ (const.) then

$$E[U_n] = \left(1 - \mathrm{O}\left(\frac{1}{n}\right)\right)\frac{n}{c+1}.$$

If $\mathfrak{d}(i) = \mathrm{o}(i)$ then $E[U_n] \xrightarrow{n\to\infty} \infty$.
(e.g., $\mathfrak{d}(i) = \lceil \sqrt{i} \rceil$, $\mathfrak{d}(i) = \lceil \log i \rceil$ etc.)

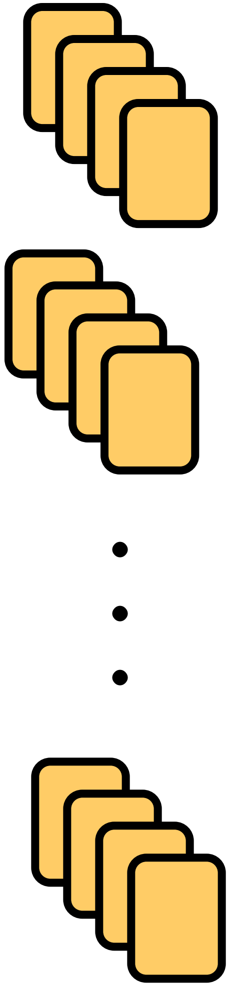
From collecting coupons
to random walks on finite graphs

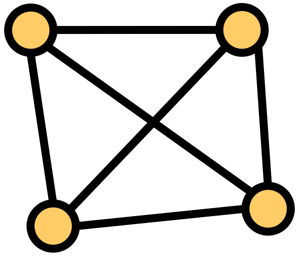
How many vertices does a random walk miss in a network *with a moderately increasing number of vertices*?

*Shuji Kijima (Shiga Univ.)
Nobutaka Shimizu (Tokyo Tech)
Takeharu Shiraga (Chuo Univ.)

in Proc. **SODA 2021**
(open access)

(Classic) collecting coupons and RW on a complete graph





completion time of coupons = **cover time** of RW
on a complete graph

Random walks on growing graphs

RWoGG: $\left(\mathfrak{d}, \left(G^{(i)}\right)_{i=1}^{\infty}, \left(P^{(i)}\right)_{i=1}^{\infty}\right)$

Thm. (general upper bound)

If $\mathfrak{d}(i) \geq c t_{\mathrm{hit}}(i)$ $(c \geq 1)$ then $\mathrm{E}[U] = O(1)$.

Particularly, if $\frac{\mathfrak{d}(i)}{t_{hit}(i)} \xrightarrow{i\to\infty} \infty$ then $\mathrm{E}[U_n] \xrightarrow{n\to\infty} 0$.

Thm. (upper bound for lazy and reversible walk)

Suppose $P^{(i)}$ is lazy and reversible.

If $\frac{t_{\mathrm{hit}}(i)}{t_{\mathrm{mix}}(i)} \geq \frac{i^{\gamma}}{c}$ and $\mathfrak{d}(i) \geq \frac{3c t_{\mathrm{hit}}(i)}{i^{\gamma}}$ $(c > 0)$ then $\mathrm{E}[U_n] \leq \frac{8n^{\gamma}}{c} + 32$.

S. Kijima, N. Shimizu, T. Shiraga, How many vertices does a random walk miss in a network with moderately increasing the number of vertices?, in Proc. SODA 2021, 106—122.

Summary

- ✓ **Collecting an increasing number of coupons.**
- ✓ Cover times of random walks on growing networks.
  - ➢ $\mathrm{E}[U_n]$ with respect to Duration $\mathfrak{d}$.

Future works

- ✓ **Does** $\mathbf{E}[\boldsymbol{U_n}] = \mathbf{O}(\mathbf{1})$ **require** $\mathfrak{d}(i) = \Omega\big(t_{\mathrm{hit}}(i)\big)$?
- ✓ Other than “cover time”?
  - Coming soon.
- ✓ From random walks to optimization:

Welcome!

- □ “Optimization in Dynamic Environment”

  Cf. calculus of variations

‘Many traditional approaches and measures for static networks are not adequate for dynamic networks. There is already strong evidence that there is room for the development of a rich theory.’

--- Othon Michail & Paul G. Spirakis CACM 2018

Fin.

The end

Thank you for the attention.